**(\*) Chapter 1**

**Problem 1:**

**The number of cars sold, S, by a dealership, is related to the advertising budget, A, by the equation:**

**S = 20A + 300.**

**(a) Illustrate this relation by plotting a graph of S against A for 0 ≤ A ≤ 15.**

**(b) Hence, or otherwise, calculate the number of:**

**(i) cars sold when the advertising budget is $8,000.**

**(ii) advertising budget required to sell 500 cars.**

**(c) State the values of the slope and intercept of the graph and give an interpretation.**

**Problem 2:**

**The revenue, R, generated by a company is related to the quantity sold, Q, by the equation:**

**R = 12Q + 500**

**(a) Illustrate this relation by plotting a graph of R against Q for 0 ≤ Q ≤ 40.**

**(b) Hence, or otherwise, calculate the:**

**(i) revenue generated when the company sells 25 units.**

**(ii) quantity required to generate $1,000 in revenue.**

**(c) State the values of the slope and intercept of the graph and give an interpretation.**

**Problem 3:**

**A bookstore finds that if it reduces the price of a novel by $2, it sells 15 more copies every week.**

**(a) Assuming that weekly sales, S, are related to prices, P, by a linear model, S = aP + b, state the value of a.**

**(b) If the bookstore sells 300 novels in a week when the price is $10, work out the value of b.**

**(c) Use this model to estimate weekly sales when the price is $8.**

**(d) If the bookstore can only sell 200 novels in a week, work out the price that it needs to charge to sell all of them.**

**Problem 4:**

**A clothing store finds that if it increases the price of a shirt by $5, it sells 10 fewer shirts every day.**

**(a) Assuming that daily sales, S, are related to prices, P, by a linear model, S = aP + b, state the value of a.**

**(b) If the clothing store sells 150 shirts in a day when the price is $20, work out the value of b.**

**(c) Use this model to estimate daily sales when the price is $25.**

**(d) If the clothing store can only sell 80 shirts in a day, work out the price that it needs to charge to sell all of them.**

**Problem 5:**

**If the demand function of a good is**

**2 P + 3 QD =60**

**Where P and QD denote price and quantity demanded respectively, find the largest and smallest values of P for which this function is economically meaningful.**

**Problem 6:**

A music store sells both CDs and digital downloads. The total annual sales of an album, in either CD or digital form, are 5000. Each CD costs $15, and each digital download costs $10. The total revenue is $70,000.

(a) If x and y represent the number of CDs and digital downloads sold, write down a pair of simultaneous equations.

(b) Solve the equations to find the number of digital downloads sold.

**Problem 7:**

A grocery store sells both fresh produce and canned goods. The total weekly sales of a specific item, in either fresh or canned form, are 1000 units. Each fresh item costs $5, and each canned item costs $3. The total revenue is $4000.

(a) If x and y represent the number of fresh items and canned items sold, write down a pair of simultaneous equations.

(b) Solve the equations to find the number of canned items sold.

**Problem 8:**

A distribution center sends three different types of parcels. One consignment has 6 small, 8 medium, and 9 large parcels which cost $173.20 to post. Another consignment has 7 small, 13 medium, and 17 large parcels with a total postage of $291.05. A large parcel costs twice as much to post as a small one. Work out the total cost of posting 3 small parcels, 9 medium parcels, and 2 large parcels.

**Problem 9:**

The demand and supply functions for a particular service are given by:

P = -3QD + 40

P = 2QS + 10

Where P, QD, and QS represent the price, quantity demanded, and quantity supplied respectively.

(a) Determine the equilibrium price and quantity.

(b) Determine the effect on the market equilibrium if the government decides to impose a fixed tax of $5 on buyers for each service. Hence, calculate the tax that the seller and buyer pay.

**Problem 10:**

The demand and supply functions for a specific product are given by:

P = -4QD + 60

P = 2QS

Where P, QD, and QS represent the price, quantity demanded, and quantity supplied respectively.

(a) Determine the equilibrium price and quantity.

(b) Determine the effect on the market equilibrium if the government decides to impose a fixed tax of $12 on each product sold. Hence, calculate the tax that the seller and buyer pay.

→New supply function: 2Qs+12,In Equilibrium,Qd=Qs=Q^'

-4Q’+60=2Q’+12→Q^'=8→P^'=$28

Tax buyer pay=P’ – P =28 – 20 =$8

→tax seller=12-8=$4

**Problem 11:**

The demand and supply functions for a specific product are given by:

P=-3Qd+60

P=2Qs+20

(a) The government imposes a tax of $t on each product sold. If the buyer has to pay $6 in tax, how much tax does the seller have to pay?

(b) The government imposes a tax of $t on buyers. If the sellers have to pay $6 in tax, how much tax do the buyers have to pay? Hence, compare this result with part (a)

**Problem 12:**

The demand and supply functions for two interdependent commodities are given by

QD1 = 100 − 2 P1 + P2

QD2 = 5 + 2 P1 − 3 P2

QS1 = −10 + P1

QS2 = −5 + 6 P2

Where QD i, QS i, and P i denote the quantity demanded, quantity supplied, and price of good i respectively. Determine the equilibrium price and quantity for this two-commodity model.

**Problem 13:**

A demand function of a certain good is given by

Q = −20 P + 0.04 Y + 4 T + 3 Pr

Where Q and P denote the quantity and price of the good, Y is income, T is taste, and

Pr is the price of a related good.

(a) Calculate Q when P = 8, Y = 1000, T = 15 and Pr = 30.

(b) Is the related good substitutable or complementary? Give a reason for your answer.

(c) Find the value of P when Q = 235, Y = 8000, T = 30 and Pr = 25.

(d) The exogenous variables are now fixed at Y = 2000, T = 10, and Pr = 5. State the values

of the slope and vertical intercept when the demand function is sketched with

(i) P on the horizontal axis and Q on the vertical axis

(ii) Q on the horizontal axis and P on the vertical axis.

**Problem 14:**

The supply and demand functions of a good are given by

P = QS + 8

P = −3QD + 80

Where P, QS, and QD denote price, quantity supplied, and quantity demanded respectively.

(a) Find the equilibrium price and quantity if the government imposes a fixed tax of $36 on each good.

(b) Find the corresponding value of the government’s tax revenue.

**Problem 15:**

The demand and supply functions of a good are given by

P = −3 QD + 60

P = 2 QS + 40

respectively. If the government decides to impose a tax of $ t per good, show that the

equilibrium quantity is given by

Q = 4-1/5 t

and write down a similar expression for the equilibrium price.

(a) If it is known that the equilibrium quantity is 3, work out the value of t. How much

of this tax is paid by the firm?

(b) If, instead of imposing a tax, the government provides a subsidy of $5 per good, find

the new equilibrium price and quantity.

**Problem 16:**

The demand and supply functions for three interdependent commodities are

QD1 = 15 − P1 + 2 P2 + P3

QD2 = 9 + P1 − P2 − P3

QD3 = 8 + 2 P1 − P2 − 4 P3

QS1 = −7 + P1

QS2 = −4 + 4 P2

QS3 = −5 + 2 P3

Where QD i, QS i, and P i denote the quantity demanded, quantity supplied, and price of good i respectively. Determine the equilibrium price and quantity for this three-commodity model.

**(\*) Chapter 2**

**Problem 1:**Given the supply and demand functions:

P =

P =

a) Determine the equilibrium price and quantity.

b) Draw a graph and show the equilibrium price and quantity

c) If government imposed a fixed tax of $80 per good sold, calculate the new equilibrium price and quantity, redraw the new graph (on the same axis as the old graph), and show the tax rates that buyers and sellers must pay. From there, indicate the seller's price and the buyer's price in the market.

**Problem 2:**

Given the supply and demand functions:

Qs =

Qd =

a) Determine the equilibrium price and quantity.

b) Draw a graph and show the equilibrium price and quantity.

**Problem 3:**

A toy manufacturer sells dolls to retailers for $120 each. If a store agrees to buy more than 20 dolls, the manufacturer is willing to reduce the unit price by $1 for each doll bought above 20, with a maximum single order of 80 dolls.

(a) How much does an order of 30 dolls cost?

(b) If the total cost of an order is $5600, how many dolls did the store buy altogether?

**Problem 4**

A company's monthly cost for paying technicians' salaries is $62,500. Under a new pay agreement, each technician earns $550 more each month. If the new pay agreement goes through, the company realizes that it will need to reduce the number of technicians by 5 if it is to cover its costs within the existing budget. What is the monthly salary of a technician before the pay increase?

**Problem 5**

A school's monthly cost for paying teachers' salaries is $84,000. Under a new pay arrangement, each teacher earns $800 more each month. If the new pay arrangement goes into effect, the school realizes that it will need to reduce the number of teachers by 6 if it is to cover its costs within the existing budget. What is the monthly salary of a teacher before the pay raise?\

**Problem 6**

Given that fixed costs are 500 and that variable costs are 10 per unit, express TC and AC as functions of Q. Hence sketch their graphs.

**Problem 7**

Given that fixed costs are 1 and that variable costs are Q + 1 per unit, express TC and AC as functions of Q. Hence sketch their graphs.

**Problem 8**

The total cost, TC, of producing 100 units of a good is 600 and the total cost of producing 150 units is 850. Assuming that the total cost function is linear, find an expression for TC in terms of Q, the number of units produced.

**Problem 9**

The total cost of producing 500 items a day in a factory is $ 40,000, which includes a fixed cost of $2000.

(a) Work out the variable cost per item.

(b) Work out the total cost of producing 600 items a day.

**Problem 10**

A taxi firm charges a fixed cost of $10 together with a variable cost of $3 per mile.

(a) Work out the average cost per mile for a journey of 4 miles.

(b) Work out the minimum distance traveled if the average cost per mile is to be less than $3.25.

**Problem 11**

Find an expression for the profit function given the demand function:

Q + P = 59

and the average cost function:

AC =

Find the values of Q for which the firm

(a) Breaks even

(b) Makes a loss of 600 units

(c) Maximizes profit.

**Problem 12**

Find an expression for the profit function given the demand function:

Q + P = 26

and the average cost function:

AC =

Find the values of Q for which the firm

(a) Breaks even

(b) Makes a loss of 500 units

(c) Maximizes profit.

**Problem 13**

A company's average cost function is given by AC =

(a) Find, to the nearest whole number, the value of Q at the lowest point on the graph of AC plotted against Q, in the interval 0 ≤ Q ≤ 40.

(b) State the value of the fixed costs.

**Problem 14**

A bakery's average cost function is given by AC =

(a) Find, to the nearest whole number, the value of Q at the lowest point on the graph of AC plotted against Q, in the interval 0 ≤ Q ≤ 50.

(b) State the value of the fixed costs.

**Problem 15**

The Ennerdale Bank charges its customers for every withdrawal: $0.50 for each online transfer and $0.25 for each cash machine withdrawal. The North Borsetshire Bank charges customers a fixed annual charge of $15 and each debit (online or machine)

costs a further $0.30. You may assume that there are no other withdrawals, that the account never goes overdrawn, and that any interest due is negligible.

(a) The proportion of withdrawals that are via online transfers is a and the total number of withdrawals made during the year is N. If the cost of operating the two accounts is the same, show that

Sketch the graph of this relationship.

(b) What advice can you of er new customers if at least 60% of the customer’s annual withdrawals are from cash machines?

**Problem 16**

The Lakeside Bank charges its customers for every withdrawal: $0.75 for each online transfer and $0.40 for each cash machine withdrawal. The Mountainview Bank charges customers a fixed annual charge of $25, and each debit (online or machine) costs a further $0.35. You may assume that there are no other withdrawals, that the account never goes overdrawn, and that any interest due is negligible.

(a) The proportion of withdrawals that are via online transfers is a, and the total number of withdrawals made during the year is N. If the cost of operating the two accounts is the same. Hence, shows that

. Sketch the graph of this relationship.

(b) What advice can you offer new customers if at least 60% of the customer’s annual withdrawals are from cash machines?

**Problem 17**

The number of complaints, N, received by a small company each month can be modeled by

N =

where t denotes the number of months since the company’s launch.

(a) Estimate the number of complaints received by the company each month for the first six months of trading.

(b) Plot a graph of N against t and hence comment on how N varies with t.

**Problem 18**

The number of customers, C, visiting a new coffee shop each month can be modeled by the equation

C = where t represents the number of months since the coffee shop's opening.

(a) Estimate the number of customers visiting the coffee shop each month for the first four months of operation.

(b) Plot a graph of C against t and comment on how the number of customers varies with time.

**Problem 19**

The population, P, of a city can be modeled by the equation P =, where t represents the number of years since a certain milestone in the city's history.

(a) Estimate the population of the city for the first three years after the milestone.

(b) Plot a graph of P against t and comment on how the population varies with time.

**Problem 20**

Show that the production function

Q =

is homogeneous and displays constant returns to scale.

**Problem 21**

(1) Define the term homogeneous when used to describe a production function f ( K, L ).

(2) If the production function

is homogeneous, state the value of m.

Does the function display decreasing, constant, or increasing returns to scale?

**Problem 22**

The value of a second-hand car reduces exponentially with age so that its value $ y after t years can be modeled by the formula

y =

If the car was $50 000 when new and was worth $38 000 after 2 years, find the values of A and “a”, correct to 3 decimal places. Use this model to predict the value of the car.

**Problem 23**

Find an expression for the profit function given the demand function

2Q + P = 25

and the average cost function

AC =

Find the values of Q for which the firm

(a) Breaks even

(b) Makes a loss of 432 units

(c) Maximises profit.

**Problem 24**

Sketch, on the same diagram, graphs of the total revenue and total cost functions,

TR =

TC = 2Q + 10

(1) Use your graphs to estimate the values of Q for which the firm

(a) Breaks even

(b) Maximises profit.

(2) Confirm your answers to part (1) using algebra.

**Problem 25**

Solve the following equations. Round your answers to 2 decimal places.

(a)

(b)

**Problem 26**

Future sales of two products A and B are given by SA =, and SB. Find the time, t, when sales of the two products are the same.

**Problem 27:**

Show that the following production function is homogeneous and state whether it displays decreasing, increasing or constant returns to scale.

**Problem 28:**

A team of financial advisers guiding the launch of a national newspaper has modelled the future circulation of the newspaper by the equation

Where N is the daily circulation after t days of publication, and c and k are positive constants. Transpose this formula to show that:

When the paper is launched, audits show that

c = 700,000 and k = ln2

(a) Calculate the daily circulation after 30 days of publication.

(b) After how many days will the daily circulation first reach 525,000?

(c) What advice can you give the newspaper proprietor if it is known that the paper will break even only if the daily circulation exceeds 750,000?

**Problem 29:**

A Cobb–Douglas production function is given by

Q =

Find an expression for **ln Q** in terms of **ln L** and **ln K**.

If a graph were to be sketched of ln Q against ln K (for varying values of Q and K but with L fixed), explain briefly why the graph will be a straight line and state its slope and vertical intercept.

**Problem 30:**

1) The demand and supply functions of a good are given by

Q D = and Q S = respectively

Where A, B, k1, and k2 are positive constants.

Find the equilibrium price and show that the equilibrium quantity is given by

**(\*2)** The total cost function of a cosmetics company is given by

Suppose if the company suddenly had to close for 2 months, how many integer values of parameter m would satisfy the company's total cost function to reach its minimum value during this closing – time?